MASTER

Solving Quadratic Equations with the Quadratic Formula

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Equations must be in the form: $ax^2 + bx + c = 0$

One side must equal zero!

Examples: use the quadratic formula to solve.

1. $3x^2 - 4x - 9 = 0$

$a=3 \quad b=-4 \quad c=-9$

$x = \frac{4 \pm \sqrt{16-4(3)(-9)}}{6}$

$x = \frac{4 \pm \sqrt{124}}{6} = \frac{4 \pm 2\sqrt{31}}{6} = \frac{\cancel{2}(2 \pm \sqrt{31})}{\cancel{6}\,3}$

$\boxed{x = \frac{2+\sqrt{31}}{3},\ \frac{2-\sqrt{31}}{3}}$

2. $2x^2 + 6x + 3 = 0$

$a=2 \quad b=6 \quad c=3$

$x = \frac{-6 \pm \sqrt{36-4(2)(3)}}{4}$

$x = \frac{-6 \pm \sqrt{12}}{4} = \frac{-6 \pm 2\sqrt{3}}{4} = \frac{\cancel{2}(-3 \pm \sqrt{3})}{\cancel{4}\,2}$

$\boxed{x = \frac{-3+\sqrt{3}}{2},\ \frac{-3-\sqrt{3}}{2}}$

3. $3x^2 - 2x + 7 = 0$

$a=3 \quad b=-2 \quad c=7$

$x = \frac{2 \pm \sqrt{4-4(3)(7)}}{6}$

$x = \frac{2 \pm \sqrt{-80}}{6}$

$x = \frac{2 \pm 4i\sqrt{5}}{6}$

$x = \frac{\cancel{2}(1 \pm 2i\sqrt{5})}{\cancel{6}\,3}$

$\boxed{x = \frac{1 \pm 2i\sqrt{5}}{3}}$

Example: Solve by factoring and using the quadratic formula.

4. $6x^2 + x - 15 = 0$

$a=6 \quad b=1 \quad c=-15$

Fact.

$(3x+5)(2x-3)=0$

$3x+5=0$ or $2x-3=0$

$3x=-5$ or $2x=3$

$\boxed{x=-\frac{5}{3} \text{ or } x=\frac{3}{2}}$

Quad Form:

$x = \frac{-1 \pm \sqrt{1-4(6)(-15)}}{12}$

$x = \frac{-1 \pm \sqrt{361}}{12} = \frac{-1 \pm 19}{12}$

$x = \frac{18}{12}$ or $\frac{-20}{12} \Rightarrow \boxed{x = \frac{3}{2} \text{ or } \frac{-5}{3}}$

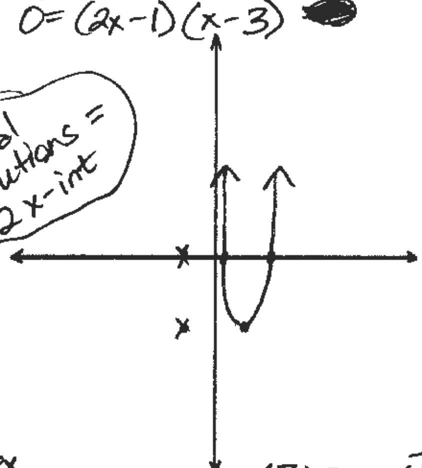
same answer

How are the solutions of a quadratic equation related to the graph of the quadratic? Graph each, then find x-intercepts.

5. $y = 2x^2 - 7x + 3$

$0=(2x-1)(x-3)$

$x=\frac{1}{2},\ x=3 \Rightarrow$ x-int.



2 real solutions = 2 x-int

vertex $x=\frac{7}{4}$

$y=2(\frac{7}{4})^2-7(\frac{7}{4})+3$

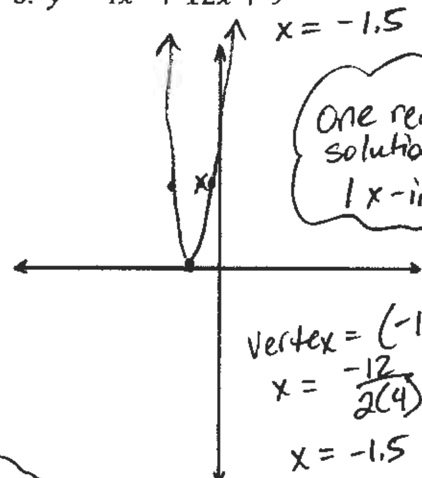
$y=-3.125$

$(1.75, -3.25)$

6. $y = 4x^2 + 12x + 9$

$x = \frac{-12 \pm \sqrt{144-4(4)(9)}}{8}$

$x=-1.5$



One real solution = 1 x-int

vertex $=(-1.5, 0)$

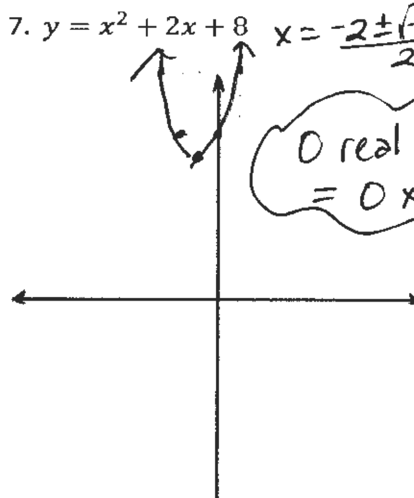
$x=\frac{-12}{2(4)}$

$x=-1.5$

7. $y = x^2 + 2x + 8$

$x = \frac{-2 \pm \sqrt{4-4(1)(8)}}{2}$

$x = \frac{-2 \pm \sqrt{-28}}{2}$



0 real solutions = 0 x-int

vertex: $x=\frac{-2}{2}=-1$

$y=(-1)^2+2(-1)+8=7$

$(-1, 7)$

Discriminant (D): Determines the number and type of Roots (Solutions)

$$D = b^2 - 4ac$$

1. *If* $D = 0$, *then there is 1 real rational repeated root.*

2. *If* $D > 0$, *then there are two real roots.*

3. *If* $D < 0$, *then there are two non − real roots.*

Examples: Describe the number and nature (real, non-real, rational) of the solutions.

8. $16x^2 + 8x + 1 = 0$

$a = 16 \quad b = 8 \quad c = 1$

$D = 8^2 - 4(16)(1)$

$D = 64 - 64 = 0$

$\boxed{\text{1 real rational solution}}$

9. $2x^2 - 5x + 6 = 0$

$a = 2 \quad b = -5 \quad c = 6$

$D = (-5)^2 - 4(2)(6)$

$D = 25 - 48 = -23$

$\boxed{\text{2 non-real solutions}}$

Application:

10. Rachel is about to serve and tosses a tennis ball straight up into the air. The height, *h*, of the ball, in meters, at time *t*, in seconds is given by $h(t) = -5t^2 + 5t + 2$. Will the ball reach a height of 4 meters?

$h(t) = -5t^2 + 5t + 2$

$4 = -5t^2 + 5t + 2$

$0 = -5t^2 + 5t - 2$

$D = b^2 - 4ac$

$D = 25 - 4(-5)(-2) = 25 - 40 = -15$

No real solutions → $\boxed{\text{No, it will not reach 4m}}$

11. Will the ball tossed in #10 reach a height of 3 meters?

$h(t) = -5t^2 + 5t + 2$

$3 = -5t^2 + 5t + 2$

$0 = -5t^2 + 5t - 1$

$D = 25 - 4(-5)(-1) = 25 - 20 = 5$

2 real solutions →

$\boxed{\text{Yes it will reach 3m 2 times (once on way up + once on way down)}}$